

The Mathematics of Innovation: Deriving Structural Art From Geometric Form

Geometric Form As A Solution

Structural innovation as a process is deeply rooted in an engineer's relationship with geometric forms. The innovative process involves many design iterations to continuously enhance a structure's performance with regard to economy, efficiency and elegance. Often, the most masterfully realized designs are those that utilize the strengths inherent within geometric form to minimize materials and construction costs, while maximizing a structure's functionality and aesthetic. These works beg the question, to what extent does the solution to innovation in structural engineering reside within the geometry itself?

¹Architects and engineers who have practiced and experimented with geometric forms are able to design with a deeper understanding of the mathematics behind those forms. Thus, the performance of the designs are enhanced. Antoni Gaudí, for instance, studied parabolic and catenary arches, which is an arch formed when the shape of a hanging rope or chain is inverted so that it stands instead of hangs. He designed the Sagrada Familia with catenary arches which allowed him to create elegant spaces that require less materials, but increase the structure's strength.



Photo 1: Sagrada Familia

In order to discuss how mastering geometric form enhances economy, efficiency and elegance, I will discuss two structures: the Montreal '67 Expo Biosphere designed by R. Buckminster Fuller, who intensively studied the mathematics behind spheres, reducing them to tetrahedrons, octahedrons and icosahedrons, and the Los Manatiales Restaurant Roof designed by Félix Candela, who utilized hyperbolic paraboloids in his design.

¹ Photo 1 by Wjh31, Wikimedia Commons

Geometric Strength

Prior to designing his geodesic dome, R. Buckminster Fuller extensively researched triangulation in tetrahedrons, octahedrons and icosahedrons, analyzing and comparing the strengths of each polyhedron. Fuller was able to determine that although tetrahedrons are especially resistant to point loads, which cause a “dimpling” effect, icosahedrons are the most efficient spatially with negligible effects on strength². He expanded his study to geodesic spheres, from which the geodesic dome was conceptualized. The intense geometrical study led to an understanding of the form as a tensegrity structure capable of enclosing a large volume while exhibiting great strength previously unknown to engineers constructing geodesic domes³. This newly identified structural strength led to a reduction in the overdesign, allowing for the innovative power of Fuller’s design to increase the efficiency and economy.



Photo 2: Montreal Expo '67 Biosphere

⁴The Montreal Expo '67 Biosphere utilizes the geodesic dome design so elegantly and timelessly that it has been featured in films and still remains an engineering marvel. The shell is light and thin while simultaneously intriguing. In 1976, a fire destroyed the acrylic shell which wrapped around the steel dome, but the structure

² Fuller, Richard Buckminster. *Synergetics: Explorations in the Geometry of Thinking*. New York: Macmillan, 1979. Sec. 612.01 and Sec. 618.30

³ Fuller, Richard Buckminster. *Synergetics: Explorations in the Geometry of Thinking*. New York: Macmillan, 1979. Sec. 794

⁴ Photo 2 by Alex Faris, Wikimedia Commons

itself withstood the force of the fire and remains intact today — 4 decades later — as a testament to the prowess of the geodesic dome.

Geometric Form For Economy

Michele Melaragno writes that Félix Candela “uses the geometry of his daring structures as a starting point that evolves through engineering and construction to completion.”⁵ His design for the Los Manantiales Restaurant Roof exemplifies the elegance permeating all facets of a structure



Photo 3: Los Manantiales Roof

when geometric sophistication is refined. The hyperbolic paraboloid vaults are arranged in a circular plan that “is emphatically not a natural form; rather, it is artificial and the product of a disciplined mind. Yet is is starkly original and obviously a work of joy.”⁶ With

a concrete shell less than 2” thick, the thinness of the structure is a direct consequence of its form. This resulting thinness could only be achieved through Candela’s continual practice with the hyperboloid paraboloid, and constant refinement of the shape. The structure is almost impossibly thin, but is strong enough that it still stands nearly six decades after its completion all the while exuding elegance, efficiency and economy.

⁵ Melaragno, Michele. *An Introduction to Shell Structures: The Art and Science of Vaulting*. Springer Science & Business Media, 2012. pp. 186.

⁶ Billington, David P. *The Tower and the Bridge: The New Art of Structural Engineering*. New York: Basic, 1983. pp. 192.

Research Proposal

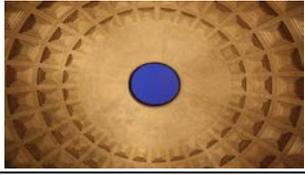
If we consider geometric form as the root of a structure, which determines its performance and functionality, the questions which arise are: how does one achieve a high quality of structural performance?; are some geometric forms better than others?; to what extent does the final form impact the structure's functionality?; how have geometrics forms evolved, and how has their evolution benefitted structures? The best way to understand how forms affect the structures is to observe them in person, where one can get a true sense of the lightness and thinness of the structures; where one can note the interaction between the structure, its environment and the humans within the space. I plan to travel to structures from different eras and different cultural backgrounds to identify unifying themes in the mathematical mindset and discover the various possible executions of structural ideas. In *The Tower and the Bridge*, David Billington makes the argument that without the need for economy, there is no structural art. It is the need to maintain cost-efficiency that drives innovation, but the dependence on economy, efficiency and elegance to become structural art.⁷ It is within this context that I will study the geometric forms, and examine how the artist was able to develop designs that are strong, functional and architecturally revered. I plan to study a variety of arches and geodesic domes in addition to parametrically designed structures, membrane structures, to name a few. Your fellowship would absolutely contribute to my growth as a structural designer by vastly expanding my understanding of what defines structural art. I would like to extend my endless gratitude for the opportunity to apply, and excitement to pursue my study around the world.

⁷ Billington, David P. *The Tower and the Bridge: The New Art of Structural Engineering*. New York: Basic, 1983. pp. 5-6

Travel Itinerary Selection

The purpose of my study is to acquire a better understanding of how geometric form drives innovation. I will select buildings and bridges of various forms, eras and cultural backgrounds to gain insight into the purpose of, and a history of the development of, forms. For the purposes of understanding how different materials may work better with different forms, I will select not only concrete and steel structures, but also wood, brick and other materials. In addition to geodesic domes and various arches, I will include membrane structures — which depend on their geometry to remain in tension — because they are extremely lightweight and have surpassed their expected lifetimes. I will also include parametrically design structures to study the ability of technological advancements to aid in the innovative process with respect to the geometric patterns of a building. Finally, I hope to visit structures which utilize their geometry for additional purposes, including sustainability and spirituality, in order to better understand the impacts of the geometric form intertwining with the nature of the structure itself.

Structure Details	Designer	Point of Interest
NORTH AMERICA		
	Montreal Expo '67 Biosphere Montreal, Quebec, Canada 1967	R. Buckminster Fuller The geodesic dome is influenced by sphere-forming icosahedrons. Photo by Abdallahh/Flickr
	<i>John Hancock Tower</i> Chicago, IL, US 1968	Arch. Fazlur Khan Eng. SOM The large diagonals brace the tubular design, helping it to support the lateral loads on the office building. Photo by Roman Boed
	<i>Gateway Arch</i> St. Louis, MO, US 1965	Eero Saarinen This is a catenary arch in its most simplistic form. Photo by Rudy Balasko
	<i>Denver International Airport</i> Denver, CO, US 1995	Arch. C. W. Fentress J. H. Bradburn Associates Eng. Severud Associates The lightweight membrane structure peaks and lulls over the terminal. Photo by Denver International Airport
EUROPE		
	<i>30 St. Mary Axe (The Gherkin)</i> London, UK 2004	Arch. Foster and Partners Eng. Arup While triangulations stiffen the structure, spirals circulate air and reduce wind load. Photo by Aurelien Guichard, Wikimedia Commons
	<i>Escuela de la Sagrada Familia</i> Barcelona, Spain 1909	Antoni Gaudí Gaudí expresses the thinness of the structure which is strengthened by its conoidal saddle-like form. Photo by Krzysztof Dydyński

	<i>Hipódromo de la Zarzuela</i> Madrid, Spain 1931	Eduardo Torroja	Doubly-curved hyperbolic surface was able to carry three times its design load, despite thicknesses of only 2" to 5.5". Photo by Peter Cresswell
	<i>Gate of Europe</i> Madrid, Spain 1996	Arch. Philip Johnson & John Burgee Eng. Leslie E. Robertson Associates	The towers are inclined at 15 deg. How does their geometric form affect the design? Photo by Bjaglin, Flickr
	<i>Saint Pierre du Vouvray Bridge</i> Normandy, France 1923	Eugene Freyssinet	When the bridge was completed, the hollow arches were the largest-spanning concrete arches in the world. Photo by Jacques Mossot
	<i>Pantheon</i> Rome, Italy 128AD	Completed under Roman Emperor Publius Aelius Hadrianus	The Pantheon is the world's largest unreinforced concrete dome. Photo by Frokor, Wikimedia Commons
	<i>Little Sports Palace</i> Rome, Italy 1957	Pier Luigi Nervi	The interior has a two-way intersecting rib pattern that stiffens the roof allowing it to be lighter. Photo by Nicolas Janberg, Structurae
	<i>M&G Research Lab</i> Venafro, Italy 1991	Arch. Samyn and Partners	The membrane structure creates an open lab space that is praised for its functionality. Photo by Samyn and Partners
	<i>Salginatobel Bridge</i> Schiers, Graubunden, Switzerland 1930	Robert Maillart	The arch bridge utilizes a hollow box to form an elegant crossing. Photo by Rama, Wikimedia Commons
	<i>Siclie Company Building</i> Geneva, Switzerland 1970	Heinz Isler	Isler incorporated the thin-shelled curve to reduce the amount of required supports for the building. Photo by Nicolas Janberg, Structurae
	<i>Olympiastadion München</i> Munich, Germany 1972	Arch. Günther Behnisch Eng. Frei Otto	The membrane structure canopies over the stadium supported by cables. Photo by M(e)ister Eiskalt, Wikimedia Commons

	<i>Zeiss I Planetarium</i> Jena, Germany 1926	Walther Bauersfeld	The structure utilizes the icosahedron to form a geodesic dome decades before the Montreal '67 Expo. Photo by Herrad Elisabeth Taubenheim
	<i>Turning Torso</i> Malmo, Skane Ian, Sweden 2005	Santiago Calatrava	Calatrava studied the geometry of the human form, observing the shape of a twisting spine for his inspiration. Photo by Sternevent GmbH
	<i>Hagia Sophia</i> Istanbul, Turkey 537AD	Built during the Byzantine Empire	First true development of pendentive form, which restrain lateral forces. Photo by Christophe Meneboeuf, Wikimedia Commons
ASIA			
	<i>The Three Pagodas</i> Dali, China 9th&10th centuries	Built under King Quan Fengyou	The pagodas survived many destructive earthquakes, despite slender form. Photo by Tdxiang, Wikimedia Commons
	<i>Shanghai Tower</i> Shanghai, China 2016	Arch. Gensler Eng. Thornton Tomasetti	The 120 degree twist in the tower produces natural wind load resistance. Photo by Baycrest, Wikimedia Commons
	<i>Petronas Twin Towers</i> Kuala Lumpur, Malaysia 1998	Arch. César Pelli & Associates Eng. Thornton Tomasetti	Design geometry reduces wind loads on the building. Photo by MithunAhamed, Wikimedia Commons
	<i>National Assembly Building of Bangladesh</i> Dhaka, Bangladesh 1982	Louis Kahn	Kahn was deeply interested in refining the geometry of his spaces and often there was a spiritual component to his spaces. Photo by Nathaniel Kahn
	<i>Kandariya Mahadeva</i> Khajuraho, India 1030	Built under Chandela King Vidhyadhara	The temple is composed of several towers encircling each other and is part of a "cosmic design of a hexagon." Photo by Atorjal, Wikimedia Commons
AUSTRALIA			
	<i>Sydney Opera House</i> Sydney, Australia 1973	Arch. Jørn Utzon Eng. Ove Arup and Partners	After many design iterations, the shells were designed as sections of a sphere. Photo by Adam J.W.C, Wikimedia Commons

	<i>Sidney Myer Music Bowl</i> Melbourne, Australia 1959	Barry Patten	This free form tensile structure was one of the earliest experiments in this form. Photo by Taylor Cullity Lethlean
SOUTH AND CENTRAL AMERICA			
	<i>Church of Cristo Obrero</i> Atlántida, Uruguay 1952	Eladio Dieste	Utilizing gaussian vaults and vertical ruled surfaces, the structure achieves thinness using brick. Photo by Julian Palacio
	<i>Cathedral of Brasilia</i> Brasilia, Brazil 1970	Oscar Neimeyer	Hyperboloid ribs are arranged in circular geometry to craft the space. Photo by Bgabel, Wikimedia Commons
	<i>Ciudad Universitaria de Caracas</i> Caracas, Venezuela 1940-1960	Carlos Raúl Villanueva	Villanueva's work brought to the city a sense of modernism that was rooted in the forms he chose. Photo by Caracas Moderna
	<i>Capilla del Gimnasio Moderno</i> Bogota, Colombia 1965	Arch. Juvenal Moya Eng. Guillermo González Zuleta	The chapel is formed of modernist paraboloid arches. Photo by lauravision.com
	<i>Los Manantiales Restaurant</i> Mexico City, Mexico 1958	Félix Candela	The hyperbolic paraboloid vaults are extremely thin, less than 2" thick. Photo by Dorothy Candela